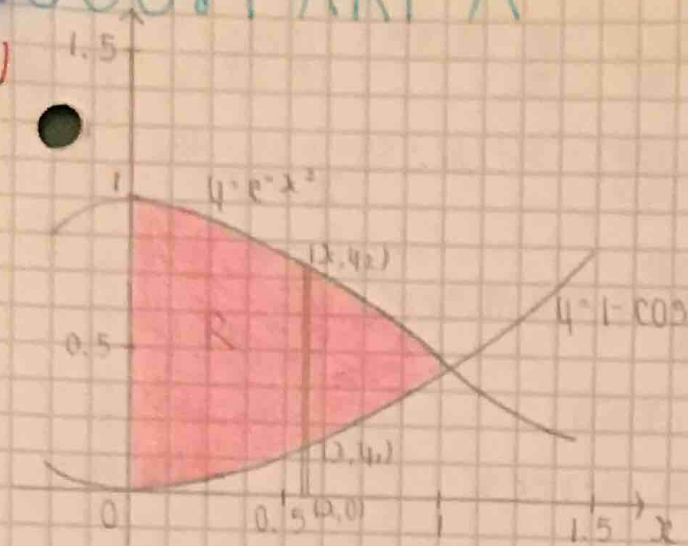
2000: PART A

1)



a) Intersection of curves:
$e^{-x^2} = 1-\cos x$
$x = 0.941944$ using G.D.C.

Area R $= \int_0^{0.941944} (e^{-x^2} - (1-\cos x))\,dx$

$= 0.591$ to 3.d.p

b) $R(x) = y_2 - 0 = e^{-x^2}$
$r(x) = y_1 - 0 = 1-\cos x$

Volume $= \pi \int_0^{0.941944} (e^{-x^2})^2 - (1-\cos x)^2\,dx$

$= 1.747$ to 3.d.p.

c)

$e^{-x^2} - (1-\cos x)$

Volume $= \int_0^{0.941944} (e^{-x^2} - (1-\cos x))^2\,dx$

$= 0.461$ to 3.d.p.

2)
a) Velocity of runner A: $y - 0 = \frac{10}{3}(x-0)$ $\therefore$ at $t=2$ $v(t) = \frac{10}{3}(2) = \frac{20}{3}$ m/sec
$y = \frac{10}{3}x$

Velocity of runner B: $v(2) = \frac{24(2)}{2(2)+3} = \frac{48}{7} = 6.857$ m/sec

b) $A(t) = V'(t)$

Acceleration of Runner A $= \frac{V(3) - V(0)}{3 - 0} = \frac{10 - 0}{3} = \frac{10}{3} = 3.333$ m/sec

Acceleration of Runner B $= V'(t) = \frac{(2t+3)(24) - 24t(2)}{(2t+3)^2} = \frac{48t + 72 - 48t}{(2t+3)^2}$

$= \frac{72}{(2t+3)^2}$

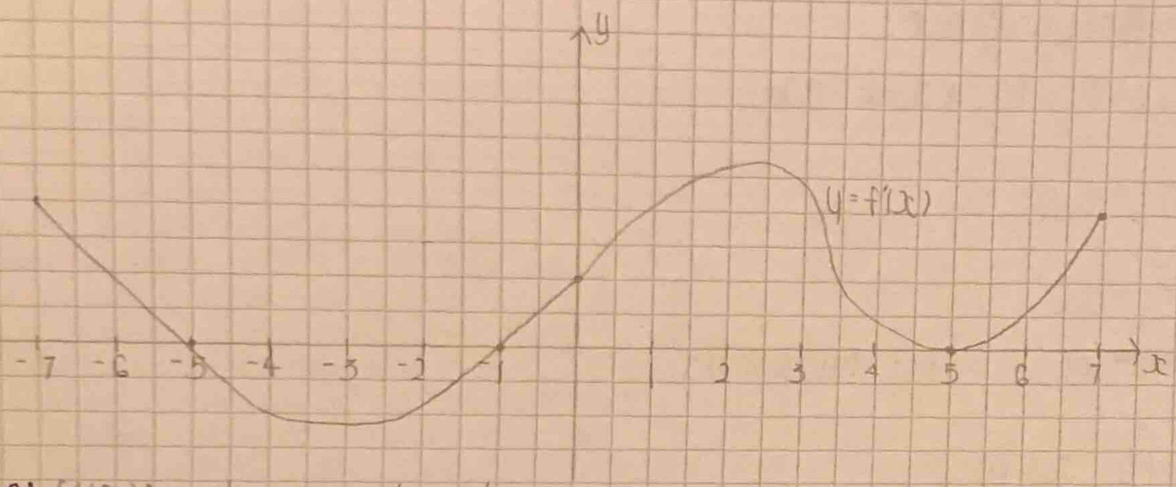
$V'(2) = \frac{72}{7^2} = \frac{72}{49} = 1.469$ m/sec^2

c) Total Distance of Runner A $= \frac{1}{2}(3)(10) + 7(10)$

$= \frac{30}{2} + 70 = 85$ meters

Total Distance of Runner B $= \int_0^{10} \frac{24t}{2t+3}\, dt = 83.356$ meters

3)



a) $f'(x)$: + on (-7, -5), − on (-5, -1), + on (-1, 5), + on (5, 7)

-7 -5 -1 5 7

Relative min when $x = -1$ since $f'(x)$ changes sign from −ve to +ve at $x = -1$

b) Relative max when $x = -5$ since $f'(x)$ changes sign from +ve to −ve at $x = -5$

c) When $f''(x) < 0$ graph is concave down. Graph is concave down when $f'(x)$ is decreasing

$(-7, -3), (2, 3), (3, 5)$

d) The absolute max must occur at $x = -5$ or at an endpoint
$f(-5) > f(-7)$ because f is increasing on $(-7, -5)$
or $f(-5) > f(-7)$ since $\int_{-7}^{-5} f'(x)dx > 0$

$\left|\int_{-5}^{-1} f'(x)dx\right| < \int_{-1}^{7} f'(x)dx \Rightarrow f(-5) < f(7)$

Absolute max at $x = 7$

000: PART B

4) a) $\int_0^3 \sqrt{t+1}\,dt$

$= \left[\frac{2}{3}(t+1)^{3/2}\right]_0^3$

$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$

$= \frac{2}{3}(8) - \frac{2}{3}$

$= \frac{16}{3} - \frac{2}{3}$

$= \frac{14}{3}$

b) $30 + \int_0^3 (8 - \sqrt{t+1}\,dt$

$30 + \int_0^3 8\,dt - \int_0^3 \sqrt{t+1}\,dt$

$30 + \left[8t\right]_0^3 - \frac{14}{3}$

$30 + 24 - \frac{14}{3}$

$\frac{148}{3}$

c) $A(t) = 30 + \int_0^t (8 - \sqrt{t+1})\,dt$

d) $A'(t) = 8 - \sqrt{t+1} = 0$

$\sqrt{t+1} = 8$

$t+1 = 64$

$t = 63$

$A'(t)$: $+$ on $(0, 63)$, $-$ on $(63, 120)$

0 63 120

/ \

Max when $t = 63$ since $A'(t)$ changes sign from +ve to -ve at $t = 63$

5) $xy^2 - x^3y = 6$

a) $y^2 + x2y\frac{dy}{dx} - \left(3x^2y + x^3\frac{dy}{dx}\right) = 0$

$y^2 + 2xy\frac{dy}{dx} - 3x^2y - x^3\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

b) $y^2 - y = 6$
$y^2 - y - 6 = 0$
$(y+2)(y-3) = 0$
$y = -2, y = 3$

$(1,-2), (1,3)$

at $(1,-2)$ $\frac{dy}{dx} = \frac{3(1)(-2) - 4}{2(1)(-2) - 1} = \frac{-10}{-5} = 2$
at $(1,3)$ $\frac{dy}{dx} = \frac{3(1)(3) - 9}{2(1)(3) - 1} = 0$

Equation of tangent @ $(1,-2)$:
$y + 2 = 2(x-1)$
$y + 2 = 2x - 2$
$y = 2x - 4$

Equation of tangent @ $(1,3)$:
$y - 3 = 0(x-1)$
$y - 3 = 0$
$y = 3$

c) When tangent is vertical, slope does not exist
$\therefore 2xy - x^3 = 0$
$x(2y - x^2) = 0$
$x = 0, \; 2y - x^2 = 0$
$x^2 = 2y$
$y = \frac{1}{2}x^2$

There is no point on the curve with x coordinate 0
When $y = \frac{1}{2}x^2$
$x\left(\frac{1}{2}x^2\right)^2 - x^3\left(\frac{1}{2}x^2\right) = 6$
$x\left(\frac{1}{4}x^4\right) - \frac{1}{2}x^5 = 6$
$\frac{1}{4}x^5 - \frac{1}{2}x^5 = 6$
$-\frac{1}{4}x^5 = 6$
$-x^5 = 24$
$x^5 = -24$
$x = \sqrt[5]{-24}$

6) $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$

a) $e^{2y}\,dy = 3x^2\,dx$
$\frac{1}{2}e^{2y} = x^3 + c$
$e^{2y} = 2x^3 + c$
$f(0) = 1/2$
$e^1 = 2(0) + c$
$c = e$

$e^{2y} = 2x^3 + c$
$2y = \ln(2x^3 + e)$
$y = \frac{1}{2}\ln(2x^3 + e)$

omain: $2x^3 + e > 0$

$x^3 > -\frac{1}{2}e$

$x > \left(-\frac{1}{2}e\right)^{1/3} = -\left(\frac{1}{2}e\right)^{1/3}$

Range: $-\infty < y < \infty$